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Enhanced resonant transmission of electromagnetic radiation through a pair of subwavelength slits

S. M. Young,1,a) C. Pfeiffer,2 A. Grbic,2 and R. Merlin1
1C-PHOM and Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1040, USA
2C-PHOM and Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, Michigan 48109-2122, USA

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We show that a pair of subwavelength slits in parallel conducting plates supports a localized electromagnetic mode bound to the slits, whose spatial extent is determined not by the plates’ size but by the slit dimensions. This mode occurs for electric fields parallel to the slits and plate separation slightly smaller than half the free-space wavelength. Finite element calculations and experimental results at 10 GHz show that the localized mode gives rise to a strong, narrowband resonant enhancement of the transmission which, while limited by conduction losses in the plates, is a factor of 104 larger than for off-resonant transmission. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4816506]

Enhanced resonant transmission of electromagnetic radiation generally transmits poorly through holes of dimension $a$ much smaller than the wavelength $\lambda$, with the ratio of transmitted power to the power incident upon the holes scaling as $(a/\lambda)^4$. However, more recent work has shown that transmission can increase dramatically if the apertures are coupled to a resonant structure. Such extraordinary transmission has been reported both for arrays of apertures coupled by surface plasmon-polaritons3–7 and for single apertures such as a narrow slit in a thick conductor exhibiting Fabry-Pérot–like resonance.8,9 A slotted metallic cavity,10 a hole surrounded by a corrugated surface,11,12 and others,13,14 Resonant cavities in particular have long been used in microwave transmission filters.15 Garcia-Vidal et al.7 provide a review of work in extraordinary transmission and its applications including sensing, near-field microscopy, and light harvesting.

In this letter, we show that the presence of a pair of subwavelength slits leads to the occurrence of a bound resonance, and thereby, to perfect transmission, even if the underlying (slit-less) structure does not support strictly localized resonances. We will consider transmission through the two-slit structure shown in Figure 1. The structure consists of two parallel conducting plates of thickness $t$, separated by a distance $d$, that extend infinitely in the $\hat{x}$ and $\hat{y}$ directions. Two parallel, narrow slits of width $w$ extend infinitely in the $\hat{y}$ direction—one in each plate. We emphasize that, in the absence of the slits, region B does not support a localized mode, as there are no walls confining the field in the $\pm \hat{x}$ directions.

We are interested in the electromagnetic power carried by waves incident from region A, through both slits, into region C. We consider incident waves that are cylindrically focused near the first slit and define the power transmission coefficient $P_t$ as the power transmitted into region C divided by the total power in the incident beam. Note that our measure of transmission is different from the area-normalized transmittance $T$ used commonly in previous work on extraordinary transmission3,7–10,16 when dealing with extended incident waves, such as plane waves. When the incident beam is cylindrically focused near the diffraction limit, the two measures are related by $P_t \sim T \lambda / \lambda$. The case $P_t = 1$ (or 0 dB) is called perfect transmission.

In the limit where the parallel plates are infinitesimally thin perfect conductors, and the slits are narrow relative to the wavelength ($2\pi w / \lambda \ll 1$), the transmission through the two-slit structure can be solved analytically by considering the effective magnetic currents induced in each slit.16,17 Previous work has primarily considered the transverse magnetic (TM) case, in which the magnetic field is parallel to the slits and transverse to the plane of propagation16,18–20 (note that this polarization is sometimes called transverse electric (TE) in the earlier engineering literature). When the incident beam is TM polarized, enhanced transmission occurs at the Fabry-Pérot–like resonance condition $d = n \lambda / 2, (n = 1, 2, …)$. However, the resonant transmission is not perfect, in part because power can be carried away from the slits via waveguide modes that propagate between the conducting plates in the $\pm \hat{x}$ directions.

From aperture theory,1,12 electromagnetic radiation generally transmits poorly through holes of dimension $a$ much smaller than the wavelength $\lambda$, with the ratio of transmitted power to the power incident upon the holes scaling as $(a/\lambda)^4$. However, more recent work has shown that transmission can increase dramatically if the apertures are coupled to a resonant structure. Such extraordinary transmission has been reported both for arrays of apertures coupled by surface plasmon-polaritons3–7 and for single apertures such as a narrow slit in a thick conductor exhibiting Fabry-Pérot–like resonance.8,9 A slotted metallic cavity,10 a hole surrounded by a corrugated surface,11,12 and others,13,14 Resonant cavities in particular have long been used in microwave transmission filters.15 Garcia-Vidal et al.7 provide a review of work in extraordinary transmission and its applications including sensing, near-field microscopy, and light harvesting.

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In this letter, we are interested in the less-studied TE case, with the electric field parallel to the slits. In this polarization, the TE\textsubscript{\textit{n}} waveguide modes that propagate between the conducting plates have cutoff frequencies given by \(f_{c,n} = \frac{n}{2d}c\), \((n = 1, 2, \ldots, )\). The analysis by Merlin\textsuperscript{17} showed that there is another mode associated with the slits that has a frequency slightly below the lowest cutoff frequency. In contrast to the TE\textsubscript{\textit{n}} waveguide modes, which extend infinitely in the \(\pm x\) directions, this additional mode is cavity-like, with the field localized near the slits as shown in Figure 2(a). The presence of this slit-bound mode, together with the poor coupling of TE radiation through the narrow slits, produces a sharp resonant peak in transmission through the pair of slits. In the limits of narrow slits and perfectly conducting plates, the resonant transmission is perfect \((P_T = 0\text{dB})\), Figure 2(b) compares the transmission coefficient for both TE and TM radiation that has been cylindrically focused on the first slit, with \(w = \lambda/10\).

To explore the properties of the localized TE mode in the presence of thick, possibly imperfectly conducting walls, we used a commercially available finite-element solver (COMSOL MULTIPHYSICS 3.5) to solve the two-dimensional (2D) Helmholtz equation

\[ \nabla^2 E_y + \epsilon_r k_0^2 E_y = 0, \]

for the geometry shown in Figure 1. To reduce the size of the numerical problem and more closely correspond to our experiment, we replaced the half-space regions A and C with rectangular waveguides excited in the TE\textsubscript{10} mode; this produces a confined incident field at the first slit that is similar to a diffraction-limited focus and results in nearly the same calculated transmission. We modeled the conducting walls as either perfect conductors \((E_y = 0)\) or using the impedance boundary condition

\[ \hat{n} \times H + \sqrt{\frac{\epsilon_r \epsilon_{r,\text{wall}}}{\mu_0}} \frac{E_y}{\beta} = 0, \]

with the permittivity given by \(\epsilon_{r,\text{wall}} = 1 + i\sigma/2\pi f \epsilon_0\), where \(\sigma\) is the conductivity and \(f\) a frequency in the microwave regime. Finally, we found that the infinite extent in the \(\pm x\) directions was best approximated by truncating the walls after several decay lengths of the localized mode and using COMSOL’s “Port” boundary condition set for the TE\textsubscript{10} mode. Figure 3 summarizes the results of the 2D numerical simulations with slit width \(w_1 = w_2 = \lambda/10\), both for perfectly conducting walls and for \(\sigma = 3.5 \times 10^3\) S/m at \(f = 10\) GHz (this value is within the range of conductivities for metals typically used in microwave waveguides\textsuperscript{21}). When the walls are perfectly conducting and very thin, the numerical transmission results agree well with the analytical solution given by Merlin.\textsuperscript{17}

For thicker—but still perfectly conducting—walls, the perfect transmission resonance is preserved and the transmission peak sharpens. Defining the quality factor \(Q = f/\Delta f\), where \(\Delta f\) is the half-power bandwidth of the transmission peak, we see \(Q\) increase from approximately \(10^3\) to \(5 \times 10^6\) when the wall thickness increases to \(\lambda/10\). This bandwidth narrowing can be understood by observing that the narrow unbeveled slits are essentially waveguides operating below the cutoff frequency; thus, increasing the wall thickness decreases the coupling exponentially between the waveguide and regions A and C. The decreased coupling is associated with narrower bandwidth and higher fields within the slotted waveguide, just as for other resonant cavities.

When the walls are made from real metals with finite conductivity, conduction losses reduce both the quality factor and the transmission. Taking losses into account, we find that increasing the wall thickness severely reduces the transmission without appreciably increasing the quality factor. If thicker walls are required for manufacturing reasons, we find that beveling the slits as shown in Figure 1 increases the transmission for a given thickness, while also decreasing the quality factor. Incorporating bevels also shifts the resonant peak to lower frequency, because the slits are effectively farther apart.

We have built and tested a two-slit structure that operates in the X-band microwave at 10 GHz, shown in Figure 4(a). The two slits are fed by commercially available rectangular WR90 waveguides, which operate in the TE\textsubscript{10}}
mode between 8.2 and 12.4 GHz. The region between the slits is milled from two pieces of stress-relieved Alloy 6061 aluminum, with polished inner surfaces. When assembled, the structure resembles a cavity with two open ends. Compared to the ideal 2D models discussed earlier, the physical structure has additional ground planes at \( y = 0 \) and \( y = h \) to accommodate the rectangular waveguide. Since the ground planes are perpendicular to the expected electric field, they are not expected to significantly perturb the field. They do, however, carry induced surface currents that lead to additional conduction losses. From symmetry considerations, we expect negligible current flow across the joint between the cavity halves. Table I summarizes the structure’s dimensions as measured after fabrication. The dimensions were chosen for ease of manufacturing while providing reasonable expected power transmission and quality factor based on the simulation results in Figure 3.

Figure 4(b) shows the power transmission coefficient through the two slits as measured using a network analyzer. The measured transmission agrees well with a three-dimensional finite element calculation that accounts for finite conductivity of the walls and additional ground planes. We used the as-built dimensions and used the conductivity of the aluminum as a fitting parameter, obtaining the best fit for \( \sigma = 1.2 \times 10^7 \, \text{S/m} \), which is consistent with other

![FIG. 3. Summary of numerical TE transmission calculations with \( w_1 = w_2 = \lambda/10 \) and various bevel angles. (a) Quality factor (i.e., inverse bandwidth) versus thickness for perfect conductors (marked with shapes) and considering losses (no shapes). (b) Maximum transmission coefficient versus wall thickness when losses are considered. (c) Effect of wall thickness on the frequency of peak transmission. This relation is the same for real metals and perfect conductors. (d) The relation between transmission coefficient and quality factor, considering losses, is the same for a variety of wall thicknesses and bevel angles.](image)

![FIG. 4. (a) Drawing of the aluminum cavity and waveguides used in the experiment. For clarity, the upper ground plane at \( y = h \) is not shown. (b) Measured (blue line) and calculated (red dots) power transmission coefficient from port 1 to port 2. (inset) Measured transmission when the open cavity ends are covered with conducting plates.](image)

![TABLE I. As-built parameters of the two-slit cavity used in the experiment.](table)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall separation</td>
<td>( d ) 14.8 mm</td>
</tr>
<tr>
<td>Slit 1 width</td>
<td>( w_1 ) 2.92 mm</td>
</tr>
<tr>
<td>Slit 2 width</td>
<td>( w_2 ) 3.07 mm</td>
</tr>
<tr>
<td>Wall 1 thickness</td>
<td>( t_1 ) 2.01 mm</td>
</tr>
<tr>
<td>Wall 2 thickness</td>
<td>( t_2 ) 2.01 mm</td>
</tr>
<tr>
<td>Slit bevel</td>
<td>( \theta ) 45°</td>
</tr>
<tr>
<td>Height in ( y )-direction</td>
<td>( h ) 10.2 mm</td>
</tr>
<tr>
<td>Length in ( x )-direction</td>
<td>( L ) 305 mm</td>
</tr>
<tr>
<td>Design operating wavelength</td>
<td>( \lambda_0 ) 30 mm</td>
</tr>
<tr>
<td>Design operating frequency</td>
<td>10 GHz</td>
</tr>
<tr>
<td>Measured peak frequency</td>
<td>( f_0 ) 10.004 GHz</td>
</tr>
<tr>
<td>Measured quality factor</td>
<td>( Q = f_0/\Delta f ) ( \approx 2760 )</td>
</tr>
<tr>
<td>Wall conductivity*</td>
<td>( \sigma ) 1.2 \times 10^7 , \text{S/m}</td>
</tr>
</tbody>
</table>

*Calculated from fit of measured transmission spectrum.
microwave conductivity measurements.\(^{21}\) The effect of conduction losses is significant even at microwave frequencies: our device has a peak transmission that is nearly two orders of magnitude lower than the ideal, perfectly conducting case. On the other hand, we observe a resonant enhancement of approximately four orders of magnitude compared to the off-resonant transmission.

The measured transmission spectrum exhibits ripples above the cutoff frequency \(f_c = c/2d\) because the walls do not extend infinitely away from the slits in the \(\pm \hat{x}\) directions but instead end after about five wavelengths. Thus, additional weak transmission peaks emerge due to propagating waveguide modes partially reflecting from the waveguide/free-space interface. If we simply cover the open ends with conducting plates, the main peak is unchanged but the additional peaks become much stronger (Figure 4(b) inset) and can be assigned to conventional cavity modes. The Fano-like\(^{22}\) shape of the additional peaks suggests interference between these cavity modes and the slit-bound mode.\(^{23}\)

In conclusion, we have demonstrated resonantly enhanced transmission associated with a localized mode bound to two narrow slits in an open structure. The effect is reminiscent of the coupling to Fabry-Pérot–like modes in open cavity devices such as masers and lasers, with the crucial difference that the apertures in our case are essential for the existence of the resonance as opposed to merely serving as a means to couple to an existing cavity mode. One consequence of the slit-bound mode is elimination of the diffraction losses usually associated with open, flat-walled cavities.\(^{24}\) The open-ended structure, with its narrow transmission peak, holds promise for applications in spectroscopy and characterization of free-flowing fluids and gases.

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